

CS 333202: Probability and Statistics

HW9 Part II

1. It follows by the memoryless property of the exponential distribution that the remaining lifetime (in thousands of miles) of the battery is exponential with parameter $\lambda = \frac{1}{10}$. Hence the desired probability is

$$P\{\text{remaining lifetime} > 5\} = 1 - F(5) = e^{-5\lambda} = e^{-1/2} \approx 0.604$$

However, if the lifetime distribution F is not exponential, then the relevant probability is

$$P\{\text{lifetime} > t + 5 \mid \text{lifetime} > t\} = \frac{1 - F(t+5)}{1 - F(t)}$$

2. Consider the time at which Mr. Smith first finds a free clerk. At this point either Ms. Jones or Mr. Brown would have just left and the other one would still be in service. However, by the lack of memory of the exponential, it follows that the additional amount of time that this other person (either Jones or Brown) would still have to spend in the post office is exponentially distributed with parameter λ . This is in it is the same as if service for this person were just starting at this point. Hence, by symmetry, the probability that the remaining person finishes before Smith must equal $\frac{1}{2}$.
3. (a) Suppose that the next customer arrives in X minutes. By the memoryless property, the desired probability is

$$P(X < \frac{1}{30}) = 1 - e^{-5(1/30)} = 0.1535$$

- (b) Let Y be the time between the arrival times of the 10th and 11th customers; Y is exponential with $\lambda = 5$. So the answer is

$$P(Y \leq \frac{1}{30}) = 1 - e^{-5(1/30)} = 0.1535$$