CS 333202: Probability and Statistics
HW9 Part I

1. $np = \frac{1095}{365} = 3$ and $\sqrt{np(1-p)} = \sqrt{3(\frac{364}{365})} = 1.73$. Therefore,

$$P(X \geq 5.5) = P(Z \geq \tfrac{5.5-3}{1.73}) = 1 - \Phi(1.45) = 0.0735$$

2. $P\{|X-3| > 6\}$
   $= P\{X > 9\} + P\{X < -3\}$
   $= P\{\frac{X-3}{3} > \frac{9-3}{3}\} + P\{\frac{X-3}{3} < \frac{-3-3}{3}\}$
   $= P\{Z > 2\} + P\{Z < -2\}$
   $= 1 - \Phi(2) + \Phi(-2)$
   $= 2[1 - \Phi(2)]$
   ≈ 0.0456

3. There are two types of errors that can occur: One is that the message 1 can be incorrectly concluded to be 0, and the other that 0 is concluded to be 1. The first type of error will occur if the message is 1 and $2 + N < 0.5$, whereas the second will occur if the message is 0 and $-2 + N \geq 0.5$. Hence

$$P\{\text{error} \mid \text{message is } 1\} = P\{N < -1.5\} = 1 - \Phi(1.5) \approx 0.0668$$

   and

$$P\{\text{error} \mid \text{message is } 0\} = P\{N \geq 2.5\} = 1 - \Phi(2.5) \approx 0.0062$$