CS 333202: Probability and Statistics
HW8 Part I

1. X is uniform on $(0, L)$.
   $P\{\min(\frac{X}{L-X}, \frac{L-X}{X}) < \frac{1}{4}\}$
   $= 1 - P\{\min(\frac{X}{L-X}, \frac{L-X}{X}) > \frac{1}{4}\}$
   $= 1 - P\{\frac{X}{L-X} > \frac{1}{4}, \frac{L-X}{X} > \frac{1}{4}\}$
   $= 1 - P\{X > \frac{L}{5}, X < \frac{4L}{5}\}$
   $= 1 - P\{\frac{L}{5} < X < \frac{4L}{5}\}$
   $= 1 - \frac{3}{5} = \frac{2}{5}$

2. $E(X) = \int_{-\infty}^{\infty} \frac{1}{2} x e^{-|x|} dx = 0$, because the integrand is an odd function. Now

   $$E(X^2) = \int_{-\infty}^{\infty} \frac{1}{2} x^2 e^{-|x|} dx = \int_0^{\infty} x^2 e^{-x} dx$$

   since the integrand is an even function; applying integration by parts to the last integral twice, we obtain $E(X^2) = 2$.
   Hence Var(X)= $2 - 0^2 = 2$.

3. Let F be the distribution function of Y. Clearly, $F(y) = 0$ if $y \leq 1$. For $y > 1$,

   $$F(y) = P(\frac{1}{X} \leq y) = P(X \geq \frac{1}{y}) = \frac{1-\frac{1}{y}}{1-0} = 1 - \frac{1}{y}.$$

   So

   $$f(y) = F'(y) = \begin{cases} \frac{1}{y^2} & y > 1 \\ 0 & \text{elsewhere} \end{cases}$$