CS 333202: Probability and Statistics
HW7 Part II

1. (a) $E[X] = \frac{1}{4}\int_0^\infty x^2 x e^{-\frac{x}{2}} dx = 2\int_0^\infty y^2 e^{-y} dx = 2\Gamma(3) = 4$

   (b) By symmetry of $f(x)$ about $x = 0$, $E[X] = 0$

   (c) $E[X] = \int_5^\infty \frac{5}{x} dx = \infty$

2. Let G and g be the probability distribution and density functions of X^2, respectively. For $t \geq 0$,

$$G(t) = P(X^2 \leq t) = P(-\sqrt{t} < X < \sqrt{t}) = F(\sqrt{t}) - F(-\sqrt{t})$$

   Thus

$$g(t) = G'(t) = \tfrac{1}{2\sqrt{t}} f(\sqrt{t}) + \tfrac{1}{2\sqrt{t}} f(-\sqrt{t}) = \tfrac{1}{2\sqrt{t}}[f(\sqrt{t}) + f(-\sqrt{t})], t \geq 0$$

   For $t < 0, g(t) = 0$

3. $E[e^X] = \int_0^\infty e^x(3e^{-3x})dx = \int_0^\infty 3e^{-2x} dx = \frac{3}{2}$