CS 333202: Probability and Statistics
HW7 Part I

1. $P(X > 15) = \int_{15}^{\infty} \frac{1}{15} e^{-x/15} dx = \frac{1}{e}$. Thus the answer is

$$\sum_{i=4}^{8} \binom{8}{i} (\tfrac{1}{e})^i (1 - \tfrac{1}{e})^{8-i} = 0.3327$$

2. (a) Since

$$1 = -\lambda(100) e^{-x/100} |_0^{\infty} = 100\lambda \text{ or } \lambda = \frac{1}{100}$$

Hence the probability that a computer will function between 50 and 150 hours before breaking down is given by

$$P\{50 < X < 150\} = \int_{50}^{150} \frac{1}{100} e^{-x/100} dx = -e^{-x/100}|_{50}^{150}$$
$$= e^{-1/2} - e^{-3/2} \approx 0.384$$

(b) Similarly,

$$P\{X < 100\} = \int_0^{100} \frac{1}{100} e^{-x/100} dx = -e^{-x/100}|_0^{100} = 1 - e^{-1} \approx 0.633$$

In other words, approximately 63.3 percent of the time a computer will fail before registering 100 hours of use.

3. The way is to derive, and then differentiate, the distribution function of Y:

$$F_Y(a) = P\{Y \leq a\} = P\{2X \leq a\} = P\{X \leq a/2\} = F_X(a/2)$$

Differentiation gives that

$$f_Y(a) = \frac{1}{2} f_X(a/2)$$