CS 333202: Probability and Statistics
HW6 Part I

1. (a) Either the Nth success should occur on the $(2N-M)$th trial or the Nth failure should occur on the $(2N-M)$th trial. By symmetry, the answer is

$$2 \cdot \binom{2N-M-1}{N-1} (\tfrac{1}{2})^N (\tfrac{1}{2})^{N-M} = \binom{2N-M-1}{N-1} (\tfrac{1}{2})^{2N-M-1}$$

   (b) The desired quantity is 2 times the probability of exactly N successes in $(2N-1)$ trials and failures on the $(2N)$th and $(2N+1)$st trials:

$$2 \binom{2N-1}{N} (\tfrac{1}{2})^N (1-\tfrac{1}{2})^{(2N-1)-N} \cdot (1-\tfrac{1}{2})^2 = \binom{2N-1}{N} (\tfrac{1}{2})^{2N}$$

2. A total of n white balls will be withdrawn before a total of m black balls if and only if there are at least n white balls in the first $n+m-1$ withdrawals. With X equal to the number of white balls among the first $n+m-1$ withdrawn balls, then X is a hypergeometric random variable, and

$$P\{X \geq n\} = \textstyle\sum_{i=n}^{n+m-1} P\{X = i\} = \sum_{i=n}^{n+m-1} \dfrac{\binom{N}{i}\binom{M}{n+m-1-i}}{\binom{N+M}{n+m-1}}$$