CS 333202: Probability and Statistics
HW5 Part II

1. Let X denote the number of games that you play, and Y the number of games that you lose.

   (a) After your fourth game you will continue to play until you lose. Therefore, $X - 4$ is a geometric random variable with parameter $1 - p$, and so

   $$E[X] = E[4 + (X - 4)] = 4 + E[X - 4] = 4 + \tfrac{1}{1-p}$$

   (b) If we let Z denote the number of losses you have in the first 4 games, then Z is a binomial random variable with parameters 4 and $1 - p$. Because $Y = Z + 1$, we have that

   $$E[Y] = E[Z + 1] = E[Z] + 1 = 4(1 - p) + 1$$

2. The probability that the station will successfully transmit or retransmit a message is $(1 - p)^{N-1}$. This is because for the station to successfully transmit or retransmit its message, none of the other stations should transmit messages at the same instance. The number of transmissions and retransmissions of a message until the success is geometric with parameter $(1-p)^{N-1}$. Therefore, on average, the number of transmissions and retransmissions is $1/(1 - p)^{N-1}$

3. Let E donate the event that the mathematician first discovers that the right-hand matchbox is empty and there are k matches in the left-hand box at the time. Now, this event will occur if and only if the $(N + 1)$th choice of the right-hand matchbox is made at the $N + 1 + N - k$ trial. Hence, we see

$$P(E) = \begin{pmatrix} 2N-k \\ N \end{pmatrix} (\tfrac{1}{2})^{2N-k+1}$$

As there is an equal probability that it is the left-hand box that is first discovered to be empty and there are k matches in the right-hand box at that time, the desired result is

$$2P(E) = \begin{pmatrix} 2N-k \\ N \end{pmatrix} (\tfrac{1}{2})^{2N-k}$$