CS 333202: Probability and Statistics
HW4 Part II

1. (a) $P\{X = i\} = i/148, i = 40, 33, 25, 50$

   $E[X] = [(40)^2 + (33)^2 + (25)^2 + (50)^2]/148 \approx 39.28$

   $E[Y] = (40 + 33 + 25 + 50)/4 = 37$

   (b) $E[X^2] = [(40)^3 + (33)^3 + (25)^3 + (50)^3]/148 \approx 1625.4$

   $Var(X) = E[X^2] - (E[X])^2 \approx 82.2$

   $E[Y^2] = [(40)^2 + (33)^2 + (25)^2 + (50)^2]/4 = 1453.5$

   $Var(Y) = 84.5$

2. with 3:

   $P\{\text{pass}\} = \frac{1}{3}[\begin{pmatrix} 3 \\ 2 \end{pmatrix}(0.8)^2(0.2)+(0.8)^3]+\frac{2}{3}[\begin{pmatrix} 3 \\ 2 \end{pmatrix}(0.4)^2(0.6)+(0.4)^3] =$
   0.533

   with 5:

   $P\{\text{pass}\} = \frac{1}{3}\sum_{i=3}^{5}\begin{pmatrix} 5 \\ i \end{pmatrix}(0.8)^i(0.2)^{5-i} + \frac{2}{3}\sum_{i=3}^{5}\begin{pmatrix} 5 \\ i \end{pmatrix}(0.4)^i(0.6)^{5-i}$
   $= 0.3038$

3. (a) $P\{h, t, t \mid 6 \text{ heads}\} = P\{h, t, t \text{ and } 6 \text{ heads}\}/P\{6 \text{ heads}\}$

   $= P\{h, t, t\}P\{6 \text{ heads}\mid h, t, t\}/P\{6 \text{ heads}\}$

   $= pq^2\begin{pmatrix} 7 \\ 5 \end{pmatrix}p^5q^2/\begin{pmatrix} 10 \\ 6 \end{pmatrix}p^6q^4$

   $= 1/10$

   (b) $P\{t, h, t \mid 6 \text{ heads}\} = P\{t, h, t \text{ and } 6 \text{ heads}\}/P\{6 \text{ heads}\}$

   $= P\{t, h, t\}P\{6 \text{ heads}\mid t, h, t\}/P\{6 \text{ heads}\}$

   $= q^2p\begin{pmatrix} 7 \\ 5 \end{pmatrix}p^5q^2/\begin{pmatrix} 10 \\ 6 \end{pmatrix}p^6q^4$

   $= 1/10$