

CS 333202: Probability and Statistics

HW4 Part I

1. An urn contains n balls, numbered 1 through n . If m balls are randomly withdrawn in sequence, each time replacing the ball selected previously, find $P\{X = k\}$, $k = 1, \dots, m$, where X is the maximum of the m chosen numbers.

Hint : First find $P\{X \leq k\}$.

2. To determine whether or not they have a certain disease, 100 people are to have their blood tested. However, rather than testing each individual separately, it has been decided first to group the people in groups of 10. The blood samples of the 10 people in each group will be pooled and analyzed together. If the test is negative, one test will suffice for the 10 people; whereas, if the test is positive each of the 10 people will also be individually tested and, in all, 11 tests will be made on this group. Assume the probability that a person has the disease is 0.1 for all people, independently of each other, and compute the expected number of tests necessary for each group. (Note that we are assuming that the pooled test will be positive if at least one person in the pool has the disease.)
3. Suppose that two teams play a series of games that ends when one of them has won i games. Suppose that each game played is, independently, won by team A with probability p . Find the expected number of games that are played when (a) $i = 2$ and (b) $i = 3$. Also show in both cases that this number is maximized when $p = 1/2$.
4. One of the number 1 through 10 is randomly chosen. You are to try to guess the number chosen by asking questions with “yes-no” answers.

Compute the expected number of questions you will need to ask in each of the two cases:

- (a) Your i th question is to be “Is it i ?”, $i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$.
- (b) With each question you try to eliminate one-half of the remaining numbers, as nearly as possible.

Hint : In (b), after 2 questions, there are 3 remaining numbers or 2 remaining numbers. If there are 3 remaining numbers, you choose 1 of the 3 and ask if that is the one.