CS 333202: Probability and Statistics
HW3 Part II

1. (a) $k(-1)^2 + k + 4k + 9k = 1 \Rightarrow k = 1/15.$

   (b) $\sum_{x=1}^{\infty} k(\frac{1}{9})^x = 1 \Rightarrow k = 1/[\sum_{x=1}^{\infty}(\frac{1}{9})^x] = 1/[\frac{1/9}{1-(1/9)}] = 8.$

   (c) $k(1+2+\cdots+n) = 1 \Rightarrow k = \frac{1}{[n(n+1)]/2} = \frac{2}{n(n+1)}.$

2. (a) Let p be the probability mass function of X and F_X be its distribution function. We have

   $$p(i) = (\tfrac{5}{6})^{i-1}(\tfrac{1}{6}), i = 1, 2, 3, \ldots$$

   $F_X(x) = 0$ for $x < 1$. If $x \geq 1$, for some positive integer n, $n \leq x < n+1$, and we have that

   $$\begin{aligned} F_X(x) &= \textstyle\sum_{i=1}^{n}(\frac{5}{6})^{i-1}(\frac{1}{6}) = \frac{1}{6}[1 + (\frac{5}{6}) + (\frac{5}{6})^2 + \cdots + (\frac{5}{6})^{n-1}] \\ &= \tfrac{1}{6} \cdot \tfrac{1-(5/6)^n}{1-(5/6)} = 1 - (\tfrac{5}{6})^n \end{aligned}$$

   (b) Let q be the probability mass function of Y. We have

   $$q(j) = P(Y = j) = P(X = \tfrac{j-1}{2}) = (\tfrac{5}{6})^{(j-3)/2}(\tfrac{1}{6}), j = 3, 5, 7, \ldots$$

3. When $\alpha > 0$

   $$P\{\alpha X + \beta \leq x\} = P\{X \leq \tfrac{x-\beta}{\alpha}\} = F(\tfrac{x-\beta}{\alpha})$$

   When $\alpha < 0$

   $$P\{\alpha X + \beta \leq x\} = P\{X \geq \tfrac{x-\beta}{\alpha}\} = 1 - \lim_{h\to 0^+} F(\tfrac{x-\beta}{\alpha} - h)$$