CS 333202: Probability and Statistics
HW2 Part I

1. Let E be the event that a 5 occurs before a 7; let F be the event that the first trial results in a 5; let G be the event that the first trial results in a 7; and let H be the event that the first trial results in neither a 5 or a 7. Conditioning on which one of these events occurs gives

$$P(E) = P(E \mid F)P(F) + P(E \mid G)P(G) + P(E \mid H)P(H)$$

However,

$$P(E \mid F) = 1$$
$$P(E \mid G) = 0$$
$$P(E \mid H) = P(E)$$

The trials are independent; therefore, the outcome of the first trial will have no effect on subsequent rolls of dice. Since $P(F) = \frac{4}{36}, P(F) = \frac{6}{36}, P(H) = \frac{26}{36}$, we see that

$$P(E) = \tfrac{1}{9} + P(E)\tfrac{13}{18}$$
$$P(E) = \tfrac{2}{5}$$

2. We have

$$P(A_1 \mid A_3 \cap A_4) = \tfrac{P(A_1 \cap A_3 \cap A_4)}{P(A_3 \cap A_4)} = \tfrac{P(A_1)P(A_3)P(A_4)}{P(A_3)P(A_4)} = P(A_1)$$

We similarly obtain $P(A_2 \mid A_3 \cap A_4) = P(A_2)$ and $P(A_1 \cap A_2 \mid A_3 \cap A_4) = P(A_1 \cap A_2)$, and finally,

$P(A_1 \cup A_2 \mid A_3 \cap A_4)$
$= P(A_1 \mid A_3 \cap A_4) + P(A_2 \mid A_3 \cap A_4) - P(A_1 \cap A_2 \mid A_3 \cap A_4)$
$= P(A_1) + P(A_2) - P(A_1 \cap A_2)$
$= P(A_1 \cup A_2)$

3. (a) 0.5

   (b) $(0.8)^3 = 0.512$

   (c) $(0.9)^7 \approx 0.4783$

   Rank : (b)(a)(c)