CS 333202: Probability and Statistics
HW1 Part II

1. (a) $P(A) = P(A \mid C)P(C) + P(A \mid C^c)P(C^c) > P(B \mid C)P(C) + P(B \mid C^c)P(C^c) = P(B)$

   (b) For the events given in the hint

   $$P(A \mid C) = \frac{P(C|A)P(A)}{3/36} = \frac{(1/6)(1/6)}{3/36} = 1/3$$

   Because $1/6 = P(A)$ is a weighted average of $P(A \mid C)$ and $P(A \mid C^c)$, it follows from the result $P(A \mid C) > P(A)$ that $P(A \mid C^c) < P(A))$. Similarly,

   $$1/3 = P(B \mid C) > P(B) > P(B \mid C^c)$$

   However,$P(AB \mid C) = 0 < P(AB \mid C^c)$

2. (a) Let A denote the event that the flashlight chosen will give over 100 hours of use, and let F_j be the event that a type j flashlight is chosen, j=1,2,3. To compute $P(A)$,condition on the type of the flashlight to obtain

   $$P(A) = P(A \mid F_1)P(F_1) + P(A \mid F_2)P(F_2) + P(A \mid F_3)P(F_3) =$$
   $$(0.7)(0.2) + (0.4)(0.3) + (0.3)(0.5) = 0.41$$

   (b) The probability is obtained by using Bayes' formula:

   $$P(F_j \mid A) = \frac{P(AF_j)}{P(A)} = \frac{P(A|F_j)P(F_j)}{0.41}$$

   Thus,

   $$P(F_1 \mid A) = (0.7)(0.2)/0.41 = 14/41$$
   $$P(F_1 \mid A) = (0.4)(0.3)/0.41 = 12/41$$
   $$P(F_1 \mid A) = (0.3)(0.5)/0.41 = 15/41$$

3. Let M, T, W, Th, F be the events that the mail is received on that day. Also, let A be the event that she is accepted and R that she is rejected.

   (a) $P(M) = P(M \mid A)P(A) + P(M \mid R)P(R)$
   $= (0.15)(0.6) + (0.05)(0.4) = 0.11$

   (b) $P(T \mid M^c) = \frac{P(T)}{P(M^c)} = \frac{P(T|A)P(A)+P(T|R)P(R)}{1-P(M)} = \frac{(0.2)(0.6)+(0.1)(0.4)}{0.89} = \frac{16}{89}$

   (c) $P(A \mid M^cT^cW^c) = \frac{P(M^cT^cW^c|A)P(A)}{P(M^cT^cW^c)}$
   $= \frac{(1-0.15-0.20-0.25)(0.6)}{(0.4)(0.6)+(0.75)(0.4)} = \frac{12}{27}$