CS 333202: Probability and Statistics
HW1

1. Let E, F, and G be three events. Find expressions for the events so that of E, F, and G:

   (a) both E and G but not F occur;

   (b) at least one of the events occurs;

   (c) all three occur;

   (d) at most one of them occurs;

   (e) exactly two of them occur;

   Hint : For example, the expression of the event that only E occurs is $E(F \cup G)^c$.

2. A certain town of population size 100,000 has 3 newspapers: A, B, and C. The proportions of townspeople who read these papers are as follows:

A: 10%	A and B: 8%	A and B and C: 1%
B: 30%	A and C: 2%	
C: 5%	B and C: 4%	

   (The list tells us, for instance, that 8,000 people read newspapers A and B.)

   (a) Find the number of people who read only one newspaper.

   (b) How many people read at least two newspapers?

   (c) How many people do not read any newspapers?

3. Which of the following statements is true? If a statement is true, prove it. If it is false give a counterexample.

   (a) If $P(A)+P(B)+P(C)=1$, then the events A, B, and C are mutually exclusive.

   (b) IP $P(A \cup B \cup C)=1$, then A, B, and C are mutually exclusive events.

4. Let $\{A_1, A_2, ...\}$ be a sequence of events. Prove that if the series $\sum_{n=1}^{\infty} P(A_n)$ converges, then $P(\bigcap_{m=1}^{\infty} \bigcup_{n=m}^{\infty} A_n) = 0$. This is called **Borel-Cantelli lemma.** It says that if $\sum_{m=1}^{\infty} P(A_n) < \infty$, the probability that infinitely many of the A_n's occur is 0.
   HINT: Let $B_m = \bigcup_{n=m}^{\infty} A_n$ and apply Theorem 1.8 to $\{B_m, m \geq 1\}$

5. A point is selected at random from the interval (0,2000). What is the probability that it is an integer?