CS 333202: Probability and Statistics
HW13 Part II

1. Cards from an ordinary deck of 52 playing cards are turned face up one at a time. If the first card is an ace, or the second a deuce, or the third a three, or ...., or the thirteenth a king, or the fourteenth an ace, and so on, we say that a match occurs. Note that we do not require that the $(13n+1)$th card be cany particular ace for a match to occur but only that it be an ace. Compute the expected number of matches that occurs.

2. Consider n independent flips of a coin having probability p of landing heads. Say that a changeover occurs whenever an outcome differs from the one preceding it. For instance, if n=5 and the outcome is H H T H T, then there is a total of 3 changeovers. Find the expected number of changeovers. Hint: Express the number of changeovers as the sum of $n-1$ Bernoulli random variables.

3. A certain region is inhabited by r distinct types of a certain kind of insect species, and each insect caught will, independently of the types of the previous catches, be of type i with probability

$$P_i, i = 1, 2, ..., r \qquad \sum_1^r P_i = 1$$

   (a) Compute the mean number of insects that are caught before the first type 1 catch.

   (b) Compute the mean number of types of insects that are caught before the first type 1 catch.

4. The positive random variable X is said to be a *lognormal* random variable with parameters μ and σ^2 if $\log(X)$ is a normal random variable

with mean μ and variance σ^2. Use the normal moment generating function to find the mean and variance of a lognormal random variable. Hint: Let $Y = log(X)$

Since Y is normal random variable, we can use the normal moment generating function $M_Y(t)$ to find the mean and variance of a lognormal random varible X.

$$M_Y(t) = E[e^{tY}]$$

Attention: Use the normal moment generating function rather than use the lognormal moment generating function.

5. Let $M_X(t) = 1/(1-t), t < 1$ be the moment-generating function of a random variable X. Find the moment-generating function of the random variable $Y = 2X + 1$.

6. For a random variable X, its moment-generating function is $M_X(t) = (1/81)(e^t + 2)^4$. Find $P(X \leq 2)$.