## CS 333202: Probability and Statistics HW13 Part I

- An absentminded professor wrote n letters and sealed them in envelopes without writing the addresses on the envelopes. Having forgotten which letter he had put in which envelope, he wrote the n addresses on the envelopes at random. What is the expected number of the letters addressed correctly?
- 2. A coin is tossed n times (n > 4). What is the expected number of exactly three consecutive heads?

**Hint:** Let  $E_1$  be the event that the first three outcomes are heads and the fourth outcome is tails. For  $2 \le i \le n-3$ , let  $E_i$  be the event that the outcome (i-1) is tails, the outcomes i, (i+1), and (i+2) are heads, and the outcome (i+3) is tails. Let  $E_{n-2}$  be the event that the outcome (n-3) is tails, and the last three outcomes are heads. Let

$$X_i = \begin{cases} 1 & \text{if } E_i \text{ occurs} \\ 0 & \text{otherwise.} \end{cases}$$

Then calculate the expected value of an appropriate sum of  $X_i$ 's.

3. Suppose that a sequence of n 1's and m 0's is randomly permuted so that each of the (n + m)!/(n!m!) possible arrangements is equally likely. Any consecutive string of 1's is said to constitute a run of 1's. For instance, if n = 6, m = 4, and the ordering is 1, 1, 1, 0, 1, 1, 0, 0, 1, 0, then there are 3 runs of 1's. Please compute the mean number of such runs.

Hint: To compute this quantity, let

$$I_i = \begin{cases} 1 & \text{if a run of 1's starts at the } i\text{th position} \\ 0 & \text{otherwise.} \end{cases}$$