## CS 333202: Probability and Statistics HW13 Part I

1. An absentminded professor wrote $n$ letters and sealed them in envelopes without writing the addresses on the envelopes. Having forgotten which letter he had put in which envelope, he wrote the $n$ addresses on the envelopes at random. What is the expected number of the letters addressed correctly?
2. A coin is tossed $n$ times $(n>4)$. What is the expected number of exactly three consecutive heads?
Hint: Let $E_{1}$ be the event that the first three outcomes are heads and the fourth outcome is tails. For $2 \leq i \leq n-3$, let $E_{i}$ be the event that the outcome $(i-1)$ is tails, the outcomes $i,(i+1)$, and $(i+2)$ are heads, and the outcome $(i+3)$ is tails. Let $E_{n-2}$ be the event that the outcome $(n-3)$ is tails, and the last three outcomes are heads. Let

$$
X_{i}= \begin{cases}1 & \text { if } E_{i} \text { occurs } \\ 0 & \text { otherwise } .\end{cases}
$$

Then calculate the expected value of an appropriate sum of $X_{i}{ }^{\prime}$ s.
3. Suppose that a sequence of $n 1$ 's and $m 0$ 's is randomly permuted so that each of the $(n+m)!/(n!m!)$ possible arrangements is equally likely. Any consecutive string of 1's is said to constitute a run of 1 's. For instance, if $n=6, m=4$, and the ordering is $1,1,1,0,1,1,0,0$, 1,0 , then there are 3 runs of 1 's. Please compute the mean number of such runs.
Hint: To compute this quantity, let

$$
I_{i}= \begin{cases}1 & \text { if a run of } 1 \text { 's starts at the } i \text { th position } \\ 0 & \text { otherwise }\end{cases}
$$

