CS 333202: Probability and Statistics
HW12 Part III

1. Since $X+Y+Z$ is Poisson with parameter $\lambda_1+\lambda_2+\lambda_3$, and $X+Z$ is Poisson with parameter $\lambda_1+\lambda_3$, we have that
$P(Y=y \mid X+Y+Z=t)$
$= \frac{P(Y=y,\, X+Z=t-y)}{P(X+Y+Z=t)}$
$= \frac{\frac{e^{-\lambda_2}\lambda_2^y}{y!}\cdot\frac{e^{-(\lambda_1+\lambda_3)}(\lambda_1+\lambda_3)^{t-y}}{(t-y)!}}{\frac{e^{-(\lambda_1+\lambda_2+\lambda_3)}(\lambda_1+\lambda_2+\lambda_3)^t}{t!}}$
$= \left(\begin{array}{c} t \\ y \end{array}\right)(\frac{\lambda_2}{\lambda_1+\lambda_2+\lambda_3})^y(\frac{\lambda_1+\lambda_3}{\lambda_1+\lambda_2+\lambda_3})^{t-y}$

2. Let X be the remaining calling time of the person in the booth. Let Y be the calling time of the person ahead of Mr. Watkins. By the memoryless property of exponential, X is exponential with parameter 1/8. Since Y is also exponential with parameter 1/8, assuming that X and Y are independent, the waiting time of Mr. Watkins, $X+Y$, is gamma with parameters 2 and 1/8. Therefore,

$$P(X+Y\geq 12) = \int_{12}^{\infty}\frac{1}{64}xe^{-x/8}dx = \frac{5}{2}e^{-3/2} = 0.558.$$

3. Let $\bar{X}$ be the average of the weights of the 12 randomly selected athletes. Let $X_1, X_2, \ldots, X_{12}$ be the weights of these athletes. Since

$$\bar{X} \sim N(225, \tfrac{25^2}{12}) = N(225, \tfrac{625}{12})$$

we have that
$P(X_1+X_2+\cdots+X_{12} \leq 2700) = P(\bar{X} \leq \frac{2700}{12}) = P(\bar{X} \leq 225) = P(\frac{\bar{X}-225}{\sqrt{625/12}} \leq \frac{225-225}{\sqrt{625/12}}) = \Phi(0) = \frac{1}{2}$