

## CS 333202: Probability and Statistics HW11 Part II

1. (a) i.  $p(0,0) = \frac{8 \cdot 7}{13 \cdot 12} = \frac{14}{39}$   
 $p(0,1) = p(1,0) = \frac{8 \cdot 5}{13 \cdot 12} = \frac{10}{39}$   
 $p(1,1) = \frac{5 \cdot 4}{13 \cdot 12} = \frac{5}{39}$
- ii.  $p(0,0,0) = \frac{8 \cdot 7 \cdot 6}{13 \cdot 12 \cdot 11} = \frac{28}{143}$   
 $p(0,0,1) = p(0,1,0) = p(1,0,0) = \frac{8 \cdot 7 \cdot 5}{13 \cdot 12 \cdot 11} = \frac{70}{429}$   
 $p(0,1,1) = p(1,0,1) = p(1,1,0) = \frac{8 \cdot 5 \cdot 4}{13 \cdot 12 \cdot 11} = \frac{40}{429}$   
 $p(1,1,1) = \frac{5 \cdot 4 \cdot 3}{13 \cdot 12 \cdot 11} = \frac{5}{143}$
- (b) i.  $P(X_1 = 1 | X_2 = 1) = \frac{4}{12} = 1 - P(X_1 = 0 | X_2 = 1)$   
ii.  $P(X_1 = 1 | X_2 = 0) = \frac{5}{12} = 1 - P(X_1 = 0 | X_2 = 0)$

2. We first obtain the conditional density of  $X$ , given that  $Y = y$ .

$$f_{X|Y}(x|y) = \frac{f(x,y)}{f_Y(y)} = \frac{e^{-x/y} e^{-y}/y}{e^{-y} \int_0^\infty (1/y) e^{-x/y} dx} = \frac{1}{y} e^{-x/y}$$

Hence

$$P(X > 1 | Y = y) = \int_1^\infty \frac{1}{y} e^{-x/y} dx = -e^{-x/y} \Big|_1^\infty = e^{-1/y}$$

3. Let  $f(x, y)$  be the joint probability density function of  $X$  and  $Y$ . Clearly,

$$f(x, y) = f_{X|Y}(x|y) f_Y(y).$$

Thus

$$f_X(x) = \int_{-\infty}^\infty f_{X|Y}(x|y) f_Y(y) dy.$$

Now

$$f_Y(y) = \begin{cases} 1 & 0 < y < 1 \\ 0 & \text{elsewhere,} \end{cases}$$

and

$$f_{X|Y}(x|y) = \begin{cases} \frac{1}{1-y} & 0 < y < 1, y < x < 1 \\ 0 & \text{elsewhere.} \end{cases}$$

Therefore, for  $0 < x < 1$ ,

$$f_X(x) = \int_0^x \frac{1}{1-y} dy = -\ln(1-x),$$

and hence

$$f_X(x) = \begin{cases} -\ln(1-x) & 0 < x < 1 \\ 0 & \text{elsewhere.} \end{cases}$$