CS 333202: Probability and Statistics
HW10 Part II

1. (a) $P(X > 1, Y < 1) = \int_0^1 \int_1^\infty 2e^{-x}e^{-2y}dxdy$
   $= \int_0^1 2e^{-2y}(-e^{-x} \mid_1^\infty)dy$
   $= e^{-1}\int_0^1 2e^{-2y}dy$
   $= e^{-1}(1 - e^{-2})$

   (b) $P(X < Y) = \int\int_{(x,y):x<y} 2e^{-x}e^{-2y}dxdy$
   $= \int_0^\infty \int_0^y 2e^{-x}e^{-2y}dxdy$
   $= \int_0^\infty 2e^{-2y}(1 - e^{-y})dy$
   $= \int_0^\infty 2e^{-2y}dy - \int_0^\infty 2e^{-3y}$
   $= 1 - \frac{2}{3}$
   $= \frac{1}{3}$

   (c) $P(X < a) = \int_0^a \int_0^\infty 2e^{-2y}e^{-x}dydx$
   $= \int_0^a e^{-x}dx$
   $= 1 - e^{-a}$

2. We start by computing the distribution function of X/Y. For $a > 0$
   $F_{X/Y}(a) = P(\frac{X}{Y} \leq a)$
   $= \int\int_{x/y \leq a} e^{-(x+y)}dxdy$
   $= \int_0^\infty \int_0^{ay} e^{-(x+y)}dxdy$
   $= \int_0^\infty (1 - e^{-ay})e^{-y}dy$
   $= \{-e^{-y} + \frac{e^{-(a+1)y}}{a+1}\} \mid_0^\infty$
   $= 1 - \frac{1}{a+1}$
   Differentiation yields that the density function of X/Y is given by $f_{X/Y}(a) = 1/(a+1)^2, 0 < a < \infty$

3. Let X and Y be the minutes past 11:30 A.M. that the man and his fiancée arrive at the lobby, respectively. We have that X and Y are

uniformly distributed over $(0, 30)$. Let $S = \{(x, y) : 0 \le x \le 30, 0 \le y \le 30\}$ and $R = \{(x, y) \in S : y \le x - 12 \text{ or } y \ge x + 12\}$. The desired probability is the area of R divided by the area of S: $324/900 = 0.36$. (Draw a figure.)

4. The problem is equivalent to the following: Two random numbers X and Y are selected at random and independently from $(0, \ell)$. What is the probability that $|X - Y| < X$? Let
$S = \{(x, y) : 0 < x < \ell, 0 < y < \ell\}$
$R = \{(x, y) \in S : |x - y| < x\} = \{(x, y) \in S : y < 2x\}$.
The desired probability is the area of R which is $3\ell^2/4$ divided by ℓ^2. So the answer is $3/4$. (Draw a figure.)