CS 333202: Probability and Statistics
HW10 Part I

1. Let X be the time until the restaurant starts to make profit; X is a gamma random variable with parameters 31 and 12. Thus $E(X) = 31/12$; that is, two hours and 35 minutes.

2. By the method of Example 5.17, the number of defective light bulbs produced is a Poisson process at the rate of $(200)(0.015) = 3$ per hour. Therefore, X, the time until 25 defective light bulbs are produced is gamma with parameters $\lambda = 3$ and $r = 25$. Hence

$$E(X) = \tfrac{r}{\lambda} = \tfrac{25}{3} = 8.33$$

3. The following solution is an intuitive one. A rigorous mathematical solution would have to consider the sum of two random variables, each being the minimum of n exponential random variables; so it would require material from joint distributions. However, the intuitive solution has its own merits and it is important for students to understand it.

   Let the time Howard enters the bank be the origin and let $N(t)$ be the number of customers served by time t. As long as all of the servers are busy, due to the memoryless property of the exponential distribution, $\{N(t) : t \geq 0\}$ is a Poisson process with rate $n\lambda$. This follows because if one server serves at the rate λ, n servers will serve at the rate $n\lambda$. For the Poisson process $\{N(t) : t \geq 0\}$, every time a customer is served and leaves, an "event" has occurred. Therefore, again because of the memoryless property, the service time of the person ahead of Howard begins when the first "event" occurs and Howard's service time begins when the second “event” occurs. Therefore, Howard’s waiting time in the queue is the time of the second event of the Poisson process $\{N(t) : t \geq 0\}$. This period, as we know, has a gamma distribution with parameters 2 and $n\lambda$.