1.

$(A-B)-C=(A-C)-(B-C)$

pf: $(A-C)-(B-C)$

$=(A\cap\bar{C})-(B\cap\bar{C})$

$=(A\cap\bar{C})\cap\overline{(B\cap\bar{C})}$

$=(A\cap\bar{C})\cap(\bar{B}\cup C)$

$=((A\cap\bar{C})\cap\bar{B})\cup\underbrace{((A\cap\bar{C})\cap C)}_{=\varnothing}$

$=A\cap\bar{C}\cap\bar{B}$

$=(A-B)\cap\bar{C}$

$=(A-B)-C.$

2.

$A-B=$ 可 $\exists x\in A-B \Rightarrow x\in A$ 且 $x\notin B \Rightarrow x\notin B \Rightarrow A-B\neq B$.
矛盾

$\Rightarrow \nexists x\in A-B \Rightarrow A-B=\varnothing=B \Rightarrow A=\varnothing.$

3. (a)

$$(A\cap B)\times(C\cap D) = (A\times C)\cap(B\times D)$$

pf:

$(a,b)\in(A\cap B)\times(C\cap D)$

$\Leftrightarrow$ $a\in(A\cap B)$ 且 $b\in(C\cap D)$

$\Leftrightarrow$ $(a\in A$ 且 $b\in C)$ 且 $(a\in B$ 且 $b\in D)$

$\Leftrightarrow$ $(a,b)\in A\times C$ 且 $(a,b)\in B\times D$

$\Leftrightarrow$ $(a,b)\in(A\times C)\cap(B\times D)$

(b)

i 當 $A=\{1\}$, $B=\{2\}$, $C=\{3\}$, $D=\{4\}$

$(A\cup B)\times(C\cup D) = (A\times C)\cup(B\times D)$ 不成立.

同學可自舉反例

ii. 取 $A=\{1,2\}$, $B=\{2\}$, $C=\{3,4\}$, $D=\{4\}$.

iii 同 ii.

4. $R=\{(a,b) \mid a-b$ is an odd positive integer $\}$.

(a)

reflexive:

$\because$ $a-a=0$ 不為正奇數

$\therefore$ $(a,a) \notin R$. No #

Symmetric:

$\forall (a,b) \in R$

$\Rightarrow a-b=2n-1$, for some $n \in N$ (其中 N 為自然數)

$\Rightarrow b-a=1-2n<0$ 不為正奇數.

$\Rightarrow (b,a) \notin R$ No #

Antisymmetric:

if $(a,b) \in R$ 且 $(b,a) \in R$.

$\because$ $(a,b) \in R \Rightarrow a-b=2n-1$. for some n, $n \in N$

$\therefore$ $(b,a) \notin R$

則 if $(a,b) \in R$ 且 $(b,a) \in R \Rightarrow a=b$

此命題為 true

$\Rightarrow$ Yes #

Transitive:

例: $a=5$, $b=2$, $c=1$. $(a,b) \in R$, $(b,c) \in R$ 但 $(a,c) \notin R$ $\Rightarrow$ No #

equivalence: 由定義及前述性質可知為 No #

(b)

reflexive : $a=a^2$，只有當 $a=1$ 時才成立

∴ R不具reflexive

No#

symmetric :

$(a,b)\in R \Rightarrow a=b^2$

if $(b,a)\in R \Rightarrow b=a^2 \Rightarrow a=a^4 \Rightarrow 1=a^3$.

只有 $a=1$ 才成立

∴ R不具 symmetric. No#

Antisymmetric :

if $(a,b)\in R$ 且 $(b,a)\in R$.

∵ 除 $a=1$ 外 $(a,b)\in R$ 且 $(b,a)\in R$

不可能成立

∴ 同(a)'s Antisymmetric.

Yes#

Transitive : ∵ if $(a,b)\in R$ 且 $(b,c)\in R$

$\Rightarrow a=b^2$ 且 $b=c^2 \Rightarrow a=c^4$.

$\Rightarrow (a,c)\notin R$

∴ R不具 Transitive. No#

equivalence : 同(a) No#

5.

reflexive : $\because$ R is an equivalence relation

$\therefore (a,a) \in R$ (R 具 reflexive)

$\Rightarrow$ 取 c 為 a

$\Rightarrow (a,a) \in S$.

symmetric : if $(a,b) \in S \Rightarrow \exists c \ni (a,c) \in R$ 且 $(c,b) \in R$

$\because$ R 具 symmetric

$\therefore (c,a) \in R$ 且 $(b,c) \in R$

$\Rightarrow (b,a) \in S$.

Transitive : if $(a,b) \in S$ 且 $(b,c) \in S$.

$\Rightarrow \exists d \ni (b,d) \in R$ 且 $(d,c) \in R$

$\because (a,b) \in S \Rightarrow \exists e \ni (a,e) \in R$ 且 $(e,b) \in R$

$\because$ R 具 Transitive $\therefore (a,b) \in R$.

$\Rightarrow (a,b) \in R$ 且 $(b,d) \in R$ 且 $(d,c) \in R$

$\Rightarrow (a,d) \in R$ 且 $(d,c) \in R$

$\Rightarrow (a,c) \in S$.

$\Rightarrow S$ is an equivalence relation

6、

$$n^3 = (n(n-1)+1)+(n(n-1)+3)+\cdots+(n(n-1)+(2n-1))$$

當 $n=1 \Rightarrow 1^3 = 1.(1-1)+1 = 1$ 成立.

假設 $n=k$ 時成立 $\left(k^3 = \sum_{i=1}^{k}(k(k-1)+(2i-1)) \right)$

考慮 $n=k+1$ 時.

$$(k+1)^3 = k^3+3k^2+3k+1$$

$$= \sum_{i=1}^{k}(k(k-1)+(2i-1)) + 3k^2+3k+1$$

$$= \sum_{i=1}^{k}(k(k-1)+(2i-1)) + 3k(k+1)+1$$

$$= \sum_{i=1}^{k}(k(k-1)+(2i-1)) + k(k+1) + 2k^2+2k+1$$

$$\left(\because k(k+1) = k(k-1)+2k \quad \therefore \sum_{i=1}^{k} k(k+1) = \sum_{i=1}^{k} k(k-1)+2k^2 \right)$$

$$= \sum_{i=1}^{k}\left[(k+1)((k+1)-1)+(2i-1)\right] + k(k+1) + 2(k+1)-1$$

$$= \sum_{i=1}^{k+1}\left[(k+1)((k+1)-1)+(2i-1)\right]$$

得証.

7.

(a) $\forall x \exists y \neg L(x, y)$

(b) $\exists x \forall y \, (L(x, y) \leftrightarrow x = y)$

8.

$\neg \exists x \forall y P(x, y) = \forall x \exists y \neg P(x, y)$

pf:

$\neg \exists x \forall y P(x, y) = \forall x (\neg \forall y P(x, y)) = \forall x \exists y \neg P(x, y)$

9. $A = \{\emptyset\}$, $B = P(P(A))$

$\Rightarrow P(A) = \{\emptyset, \{\emptyset\}\}$

$\Rightarrow P(P(A)) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\} = B$

所以 all Yes.

10. 舉 $A=\{1,2\}$, $B=\{2,3\}$ 之反例

11.

(a) $E \subseteq (A \cup B)$

(b) $(A \cap C) \subseteq D$

(c) $(B \cap \overline{C}) \subseteq \overline{D}$

12.

If $\forall a \in A\ \exists\ b \in A \ni (a,b) \in R$.

$\Rightarrow$ ∵ R 具 symmetric ∴ $(b,a) \in R$.

又 R 具 Transitive ∴ $(a,b) \in R$ 且 $(b,a) \in R \Rightarrow (a,a) \in R$ $\forall a \in A$

$\Rightarrow$ R 具 reflexive.

$\Rightarrow$ R is an equivalence relation

13. 令 p: "There is gold on the island"

q: "I always lie"

① 如果 native tell the truth 則 $p \leftrightarrow q$ 為 True 又 $\sim q \Leftrightarrow \sim p$
∴ no gold

② 如果 native lie ∵ 則 $p \leftrightarrow q$ 為 false 又 ∵ $q \to \sim p$ ∴ no gold