- 1. Let M(x, y) be "x has sent y an e-mail message" and T(x, y) be "x has telephoned y," where the universe of discourse consists of all students in your class. Use quantifiers to express each of these statements. (Assume that all e-mail messages that were sent are received, which is not the way things often work.)
  - a) Chou has never sent an e-mail message to Koko.
  - b) Arlene has never sent an e-mail message to or telephoned Sarah.
  - c) Jose has never received an e-mail message from Deborah.
  - d) Every student in your class has sent an e-mail message to Ken.
  - e) No one in your class has telephoned Nina.
  - f) Everyone in your class has either telephoned Avi or sent him an e-mail message.
  - g) There is a student in your class who has sent everyone else in your class an e-mail message.
  - h) There is someone in your class who has either sent an e-mail message or telephoned everyone else in your class.
  - i) There are two students in your class who have sent each other e-mail messages.
  - j) There is a student who has sent himself or herself an e-mail message.
  - k) There is a student in your class who has not received an e-mail message from anyone else in the class and who has not been called by any other student in the class.
  - Every student in the class has either received an e-mail message or received a telephone call from another student in the class.
  - m) There are at least two students in your class such that one student has sent the other e-mail and the second student has telephoned the first student.
  - n) There are two students in your class who between them have sent an e-mail message to or telephoned everyone else in the class.
- 2. Determine the truth value of each of these statements if the universe of discourse of each variable consists of all real numbers.

a) 
$$\forall x \exists y(x^2 = y)$$
  
b)  $\forall x \exists y(x = y^2)$   
c)  $\exists x \forall y(xy = 0)$   
d)  $\exists x \exists y(x + y \neq y + x)$   
e)  $\forall x(x \neq 0 \rightarrow \exists y(xy = 1))$   
f)  $\exists x \forall y(y \neq 0 \rightarrow xy = 1)$   
g)  $\forall x \exists y(x + y = 1)$   
h)  $\exists x \exists y(x + 2y = 2 \land 2x + 4y = 5)$   
i)  $\forall x \exists y(x + y = 2 \land 2x - y = 1)$   
j)  $\forall x \forall y \exists z(z = (x + y)/2)$